Robust active control against wind-induced structural vibrations

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Abstract

Slender flexible structures are vulnerable to vibrations under wind loads. The dynamic model of a frame-like structure is obtained by finite element approximation in this paper and used further for the design of an active control mechanism. The behavior of the structure is described by simplified linear equations. A linear quadratic regulator and an $H_2$ optimal control method are used for the suppression of the extended vibration effects. Structured uncertainties are considered to reflect the errors between the model and the reality. To accommodate directly the plant uncertainties and to obtain a best possible performance in the face of uncertainties a robust $H_{\infty}$ optimal control for active control structure is used. The two latter robust controllers take into account as well incompleteness of the measured information, a fact that cannot be neglected in civil engineering, and lead to applicable designs of smart structures. The numerical simulation shows that vibrations can be suppressed by means of the proposed methods.

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1. Introduction

Maintaining the safety and functionality of buildings, the living comfort of the residents and the functionality of sensitive equipment during earthquakes and strong winds is of paramount importance in civil engineering. The classical (passive) design is based on appropriate choice of the materials and of the dimensions of the structure. In most cases this conservative approach leads to unnecessarily heavy structures or cannot achieve the design goals. The ideas of integrating the automatic control concept or installing a mechanical control device into the seismic structural design were proposed by Kobori and Minai (1960) more than 40 years ago. In an active structure an embedded, suitable control system, can modify the internal stresses, the mass or the damping of the dynamical system and can keep the response of the resulting “intelligent” structure within required limits. An overview is given, among others, by Housner and Bergman (1997). Several control techniques have been developed. Passive control systems in the form of energy-absorbing devices are proposed, but they have a limited effectiveness. The application of active control or mixed (hybrid, or semi-active) schemes is feasible, due to recent developments in information science and computer technology. Preumont (1997) discusses some of the developed active control systems, which are based on the general theory of control for dynamical systems.

Displacement or velocity feedback is the most common active control method. It uses some structural response information and produces a set of control forces that influence the dynamical response of the building structure. One of the powerful principles here is the optimal control, in which the operational quantities are determined such as to optimize a given evaluation function. For a recent discussion of some new trends in optimal structural control the reader may consult Arvanitis et al. (2003). Linear quadratic optimal control (LQR) is used in the majority of the published works. An advantage of the LQR is the linearity of the control law, which simplifies both the analysis and the practical implementation. Other advantages include the good disturbance rejection and tracking and good stability properties. However, the classical form of LQR does not address practical issues such as system’s uncertainties. To deal with this problem robust control techniques have been developed. The optimization problem takes into account the worst distribution of uncertainty, within some given class of admissible representations of the uncertainty either of the structural model or of the external disturbance (loading). The importance of robust control in civil engineering is obvious, since the appearance of uncertainties is unavoidable. In fact, a structural model does not reproduce some details of the structural system (e.g., higher vibrational modes), while the loading is rarely defined accurately.

In the present work a dynamic model of a building subjected to earthquake- and wind-type excitations is studied. The structure is presented as a two-dimensional frame and is modeled by means of finite elements. The regulator problem is posed as an optimal control problem. Three types of cost functions LQR, $H_\infty$ and $H_2$ are considered. Classical LQR method is based on a quadratic cost functional minimizing both the kinetic energy of the structure and the control energy. The $H_2$ optimal control strategy operates with uncertainties of the system’s model and exogenous inputs to which the structure is subjected and is based on the minimization of the $L_2$ weight norm of the transfer function from the disturbances to the regulated states. Furthermore, structured uncertainties are introduced to reflect the errors between the model and the reality. In this case robust $H_\infty$
optimal control, which is based on the minimization of the $H_{\infty}$ norm of the transfer function from the perturbations to the controlled outputs, is used. A model loading, which is taken to be equal to either a periodic impulsive force or a sinusoidal one, is considered as an application. Numerical results show the effectiveness of the proposed methods.

2. Problem formulation

Simple linear plant models, which may be obtained through linearization and simplification of the true nonlinear models, are usually considered for the design of the control systems in structural analysis. Our interest is focused on time-invariant, finite dimensional dynamical systems. Let us consider a linear building structure with $n$ degrees of freedom subjected to a time-dependent external loading. In this paper the system remains linear during vibrations. The equation of motion, after finite element discretization, can be expressed in a canonical first-order state space form as (see, for instance, Yang et al., 1994)

$$x' = Ax + B_1w + B_2u. \quad (1)$$

Here $x$ denotes the state vector, which is composed from nodal displacements and velocities and $u$ is the control vector. The vector $w$ represents all disturbances of the system (dynamic excitation loads, sensor noise, model inaccuracies). Both $u$ and $w$ comprise the input vector of the system. The matrix $A$ is the state space system matrix and depends on nominal mass $M$, damping $C$ and stiffness $E$ matrices of the structure. The matrices $B_1$ and $B_2$ determine the allocations of the loading and control forces.

In an active feedback control system the signals sent to the control actuators are functions of the response of the system, which, in turn, is measured by appropriate sensors. Let us denote by $y$ the measurements, which depend on the state variables and the disturbances

$$y = C_2x + D_{21}w. \quad (2)$$

A second group of the plant outputs, $z$, denotes the controlled outputs. These are all the signals we are interested in controlling. Let us suppose that vector $z$ is a linear combination of the state variables, the disturbances and the control

$$z = C_1x + D_{11}w + D_{12}u. \quad (3)$$

The matrix $C_1$ in Eq. (2) is the regulated matrix and the matrix $C_2$ in Eq. (3) is the observation matrix. The measurements $y$ and the regulated outputs $z$ are completed with the information about control and exogenous inputs by the allocation matrices $D_{11}$, $D_{12}$ and $D_{21}$, respectively. Eqs. (1)–(3) are the plant representation in state space form of the building structure.

The nominal plant representation in state space form is

$$\begin{bmatrix} x' \\ y \end{bmatrix} = G_n \begin{bmatrix} x \\ u \end{bmatrix}, \quad (4)$$

where $G_n$ is the nominal plant

$$G_n = \begin{bmatrix} A & B_2 \\ C_2 & 0 \end{bmatrix}. \quad (5)$$
The objective in this study is to determine the optimal vector of active control forces $u(t)$ subjected to some performance criteria and satisfying the dynamical equations (1)–(3) of the structure, such as to reduce in an optimal way the external excitations.

3. Optimal control design

For the solution of the outlined regulator problem for the linear, time-invariant model, we consider the steady state (infinite time) case, i.e. the optimization horizon is allowed to extend to infinity.

3.1. Classical LQR control strategy

The plant (1) of the structure is a linear system in state space form. The matrix $A$, which describes the dynamical system, is not necessarily supposed to be stable, therefore mechanisms in light-weight coverings or facades are tractable, although they are not common in classical civil engineering applications. In this section we consider the nominal system (4) and (5). The pair $(A, B_2)$ is assumed to be stabilizable. We consider the regulator problem and seek linear state feedback control law of the form

$$u = -Kx,$$  \hspace{1cm} (6)

such that the following quadratic cost functional is minimized:

$$J = \frac{1}{2} \int_0^\infty (x^TQx + u^TRu) \, dt \rightarrow \min,$$  \hspace{1cm} (7)

where $Q \geq 0$ ($Q = Q^T$) is the state weight matrix and $R > 0$ ($R = R^T$) is the control weight matrix. The restrictions on the weight matrices imply that some of the states may be irrelevant for the problem and that the control energy must be finite. The problem (4), (5) and (7) is known as the linear quadratic regulator (LQR) problem and belongs to the powerful machinery of the optimal control. Its solution, according to Eq. (6), is a linear controller with constant gain

$$K_{LQR} = R^{-1}B_2^TP,$$  \hspace{1cm} (8)

where the constant matrix $P$ is a solution of the algebraic Riccati equation (ARE)

$$A^TP + PA + Q - PB_2R^{-1}B_2^TP = 0.$$  \hspace{1cm} (9)

Under technical assumptions, which can be found in classical control theory treatises, existence and uniqueness of the controller (8) asymptotically stabilizing the system is guaranteed. The main design parameters for the controller are the weight matrices $Q$ and $R$.

It can be shown, for example Shahian and Hassul (1994), that from the Return difference inequality and Small gain theorem follows that the optimal system will have good feedback properties, namely, good disturbance rejection and tracking. An advantage of the LQR solution of the problem is the linearity of the control law, which simplifies the analysis and the practical implementation. LQR solution requires that all states are available for feedback and represents a state feedback problem. LQR is designed to satisfy specified requirements for steady state error, transient response, stability margins, or closed loop pole location. It has attractive properties that, unfortunately, are lost when the...
regulator has to use unavailable (unmeasured, usually estimated by means of some filter) states. In addition LQR does not address model uncertainty.

3.2. Robust $H_2$ control strategy

Too much emphasis on optimality and less attention to the model uncertainty leads to control that fail to work in real environments. This fact, together with the lack of user-friendly software, certainly contributed to the restricted use of automatic control in civil engineering. Robustness with respect to external disturbances or uncertainties of the system or of the loading is the key issue, especially for civil engineering applications. Two popular methods, $H_2$ and $H_\infty$ optimal control strategies, can be applied for these purpose. They use as performance measures the $H_2$ and $H_\infty$ norms of the stable transfer matrix in the frequency domain. $H_2$ optimal control is studied in this section with the assumption that the exogenous signals are fixed or have fixed power spectrum.

Consider the system Eqs. (1)–(3). The inputs $w$, $u$ and the outputs $y$, $z$ are continuous-time signals. After Laplace transformation the realization of the plant transfer matrix $G$ in the frequency domain is taken to be of the form:

$$ G(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix}. $$

The corresponding two-port block diagram is shown in Fig. 1, where $K$ is the controller that is to be determined. Both $G$ and $K$ are supposed to be real, rational and proper. We search the controller in the linear feedback form (4) that will keep the regulated outputs $z$ as small as possible under the existence of the exogenous inputs $w$. Let $T_{zw}$ denote the transfer function from $w$ to $z$. We get as a performance criterion the minimization of the $H_2$ norm of $T_{zw}$, i.e.

$$ \| T_{zw} \|_2 = \left( \frac{1}{2} \int_{-\infty}^{+\infty} \text{trace} [T_{zw}(j\omega) T_{zw}^*(j\omega)] d\omega \right)^{1/2}, \quad (10) $$

over all internally stabilizing the system controllers $K$. One way to explain the mechanical meaning of this criterion is the following. Let $m$ denotes the dimension of $w$ and denote by $e_i$, $i = 1, m$, the standard basis in $\mathbb{R}^m$. An impulse at the $i$th component of the exogenous signal is achieved by setting $w(t) = \delta(t)e_i$, the resulting output being $z_i(t) = T_{zw}(t)e_i$. Then it is easy to derive that

$$ \| T_{zw} \|_2 = \left( \sum_i \| z_i \|_2^2 \right)^{1/2} = \left( \sum_i \| T_{zw} \delta e_i \|_2^2 \right)^{1/2}. \quad (11) $$

![Fig. 1. Closed-loop system diagram for robust $H_2$ control.](image)
The $H_2$ norm of $T_{zw}$ minimizes the worst-case root-mean square value of the regulated variables when the disturbances are white processes of intensity equal to one. In this case the calculation of a state space solution to the frequency domain optimization problem is possible. Under some assumptions that are explained by Zhou (1998) it can be shown that there exists a unique controller $K_2$ which minimizes $T_{zw}$ with the following transfer matrix representation:

$$K_2 = \begin{bmatrix} A - B_2B_2^T X - YC_2^T C_2 & YC_2^T \\ -B_2^T X & 0 \end{bmatrix},$$

where $X$ and $Y$ are the solutions of the two ARE

$$A^T X + XA - XB_2B_2^T X + C_1^T C_1 = 0, \quad AY + YA^T - YC_2^T C_2 T + B_1B_1^T = 0, \quad (12)$$

for a stable matrix $A$. The controller $K_2$ has a separable structure. From the measurements, the whole system is first reconstructed in an optimal way using Kalman–Bucy filter during the estimation phase, and then the optimal control problem is based on this reconstructed state vector.

### 3.3. Robust $H_{\infty}$ control strategy

The nominal model parameters of the mechanical system $\bar{M}$, $\bar{C}$ and $\bar{E}$ are determined by the material properties and geometry configuration using classical finite element techniques. Since the physical parameters of the real structure system are not known exactly, we take a mean value for the determination of the nominal values and all other deviations are included in the uncertainty of the model. The exogenous influences acting on the system lead to errors in tracking. Parameter perturbations in the system can amplify significantly the effect of these disturbances. Thus, the appearance of model parameter uncertainties is a common task in building structural control. For several reasons it is highly desirable to introduce structured uncertainties for the physical parameters of the system. To obtain a best possible performance in the face of the uncertainties, a robust $H_{\infty}$ optimal control for active control structures is considered in this section. The implementation of $H_{\infty}$ control theory is motivated by the inability of the $H_2$ theory to directly accommodate plant uncertainties. The robust control design will be formulated here within the framework of linear fractional transformation (LFT) (Zhou, 1998), which is particularly useful in the study of perturbations.

Let us consider the nominal plant (4) and (5) and suppose that the three actual physical parameters $M$, $C$ and $E$ are not known exactly, but are believed to lie in known intervals. In particular, the actual mass $M$ is within $p_M$ percentage of the nominal mass $\bar{M}$, the actual damping value $C$ is within $p_C$ percentage of the nominal value $\bar{C}$, and the spring stiffness $E$ is within $p_E$ percentage of its nominal value of $\bar{E}$. Now introducing real perturbations

$$\Delta_M = \delta_M I, \quad \Delta_C = \delta_C I, \quad \Delta_E = \delta_E I, \quad (13)$$

which are assumed to be unknown but restricted

$$-1 \leq \delta_M, \delta_C, \delta_E \leq 1, \quad (14)$$

we can write the actual physical parameters of the system in the following form:

$$M = \bar{M}(I + p_M \Delta_M) \quad C = \bar{C}(I + p_C \Delta_C) \quad E = \bar{E}(I + p_E \Delta_E). \quad (15)$$
The uncertainty in the matrices $M^{-1}$, $C$ and $E$ can be represented by the LFT matrix functions as upper LTF in the perturbations $\Delta_M$, $\Delta_C$ and $\Delta_E$

$$M^{-1} = F_U \begin{bmatrix} -p_M I & M^{-1} \\ -p_M I & M^{-1} \end{bmatrix}, \quad C = F_U \begin{bmatrix} 0 & \bar{C} \\ p_C I & \bar{C} \end{bmatrix}, \quad E = F_U \begin{bmatrix} 0 & E \\ p_E I & E \end{bmatrix}. \quad (16)$$

To represent the model as LFT of the uncertainty parameters $\delta_M$, $\delta_C$, $\delta_E$, we first isolate the uncertainty parameters and denote the inputs of $D_M$, $D_C$, $D_K$ as $y_M$, $y_C$, $y_E$ and their outputs as $u_M$, $u_C$, $u_E$. The outputs $u_A = [u_M u_C u_E]^T$ from the perturbations are added to the system’s inputs. The inputs $y_A = [y_M y_C y_E]^T$ to the perturbations are added to the system’s outputs. The model for the uncertain system is obtained in the following matrix form:

$$\begin{bmatrix} x' \\ y_A \\ y \\ u \\ u_A \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} x \\ u_A \\ u \end{bmatrix} = A y_A, \quad (17)$$

where $G$ is the plant of the perturbed system

$$G = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 & 0 \\ -p_M I & -p_C M^{-1} & -p_E M^{-1} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -EM^{-1} & -CM^{-1} \\ 0 & C \\ E & 0 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} -p_M I & -p_C M^{-1} & p_E M^{-1} \\ 0 & 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} H M^{-1} \\ 0 \end{bmatrix}, \quad D_{21} = [0 \ 0 \ 0]. \quad (18)$$

The matrix $G$ of the system (17) contains only physical nominal parameters of the system and, therefore, it is known. The system model uncertainty matrix in Eq. (17), denoted by $A$, is a matrix with the following block diagonal structure:

$$A = \begin{bmatrix} \Delta_M & 0 & 0 \\ 0 & \Delta_C & 0 \\ 0 & 0 & \Delta_E \end{bmatrix}.$$

This latter matrix influences the input/output connection between the control $u$ and the output $y$ in a way that can be represented as a feedback by the upper LFT

$$y = F_U(G, A)u. \quad (19)$$

The system matrix $G$ describes the set of perturbed models. For the robust stability analysis the controller $K$ can be viewed as a known system component and absorbed into
an interconnection structure \( P \) together with the plant \( G \). For the robust stability we are interested in finding the smallest perturbation \( \Delta \), real and norm bounded \( \| \Delta \|_\infty < 1 \) in the sense of maximal singular value \( \sigma(\Delta) \), which destabilizes the closed loop framework. The loop is well-posed and internally stable for all \( \Delta \) with \( \| \Delta \|_\infty < 1 \) if and only if

\[
\sup_{\omega \in \mathbb{R}} \mu_\Delta(p(j\omega)) < 1,
\]

where \( \mu_\Delta \) is the structured singular value. Therefore, the quantity

\[
\frac{1}{\max_{\omega} \mu_\Delta(P(j\omega))},
\]

is a stock of stability with respect to the structured uncertainty influenced \( P \).

The exogenous influences, which act on the system, lead to errors in tracking. Parameter perturbations in the system can amplify significantly the effect of these disturbances. As a result the performance of the closed-loop system deteriorates and eventually loses stability. That is the reason for trying to ensure robust performance for a given level of perturbations. The performance criterion is to keep the errors as small as possible for all perturbed models. The performance specifications will be specified on the closed-loop frequency response of the transfer matrix between the disturbances and the errors, which places the problem into the framework of \( H_\infty \) design. In the case of a system with uncertainty a convenient characteristic for the robust performance is the sensitivity or complementary sensitivity transfer matrix (or their combination) from the exogenous disturbances to the errors. For our purposes we take the sensitivity transfer matrix of the closed-loop system as a performance criterion and we will require for its norm the following inequality:

\[
\| W_p(I + GK)^{-1} \|_\infty < 1,
\]

to arise a desirable performance. This requirement must be satisfied for a given weight matrix \( W_p \) such that to reject the disturbances for low frequencies at the output.

The linear system (17) can be described with the flow diagram of Fig. 2.

The uncertainty block \( \Delta \) is supposed to be stable and norm bounded \( \| \Delta \|_\infty \leq 1 \). The block \( K \) is the controller that has to be determined. The nominal plant \( G_n \) has three sets of inputs: uncertainty inputs \( u_\Delta \), external disturbances \( d \) and control commands \( u \). Three sets of outputs are generated: uncertainty outputs \( y_\Delta \), errors \( e \) and measurements \( y \).

Fig. 2. Closed-loop system diagram for robust \( H_\infty \) control.
Let \( w = [u_d d]^T \) are all external inputs coming to the system and \( z = [y_d e]^T \) are all signals characterizing system’s behavior. Then the system (17) is transformed in

\[
\begin{bmatrix}
  z \\
y
\end{bmatrix} = \begin{bmatrix}
w \\
u
\end{bmatrix} \quad u = K(s)y.
\]

(23)

The transfer matrix \( G \) contains weights for the uncertainty, which depends on the control design. The closed-loop system’s transfer matrix from \( w \) to \( z \) is given by a lower LFT

\[ z = F_L(G, K)w. \]

(24)

The problem of the control design consists in the determination of a controller \( K_{\text{inf}} \) that ensures internal stability of the system (4) and keeps the transfer matrix \( F_L(G, K) \) between \( w \) and \( z \) minimal in the sense of the \( H_\infty \) norm.

\[
\| F_L(G, K_{\text{inf}}) \|_\infty = \max \{ \tilde{\sigma}(F(G, K)(j\omega)) \} \rightarrow \min.
\]

(25)

The transfer matrix \( F_L(G, K) \) contains measures of the nominal performance and the stability robustness. Its \( H_\infty \) norm gives a measure of the worst-case response of the system over an entire class of input disturbances. The optimal \( H_\infty \) controller is not unique for our MIMO system (in contrast with the standard \( H_2 \) theory, in which the optimal controller is unique). The knowledge of the optimal \( H_\infty \) norm is useful, in principle, since it sets a limit on what the controlled system can achieve. In practice, it is often not necessary to design an optimal controller and it is much cheaper to obtain a controller that is close, in the norm sense, to the optimal one. For a given \( \gamma > 0 \), we will find an admissible suboptimal controller \( K_{\text{sub}}(s) \) such that the \( H_\infty \) norm closed-loop transfer matrix of the system (24) from \( w \) to \( z \) is less than \( \gamma \).

\[
\| F_L(G, K_{\text{sub}}) \|_\infty < \gamma.
\]

(26)

4. Numerical simulations

In order to demonstrate the validity of the proposed methodologies, a computer simulation is carried out on a model, two-dimensional eight-storey building structure. The objective is to use the LQR, \( H_2 \) and \( H_\infty \) optimal active controls to suppress the wind-induced vibrations. The horizontal elements with length of 10 m represent the floors of the building while the vertical ones with height of 3 m represent the columns. Every element has two nodes and every node has three degrees of freedom (d.o.f.)—horizontal displacement, vertical displacement and rotation, respectively. The structure has 22 members and 16 nodes (excluding supports). This results in 96 state variables.

The placement of measurements and controllers has a paramount influence on the effectiveness of the control scheme. The number of possible combinations for the placement of sensors and controllers in an eight-storey building is very large. Let four sensors are placed in the nodes of the first-, third-, fifth- and seventh-floor of the structure and measure the horizontal displacements of the corresponding node. We propose to place four actuators in the nodes of the same floors of the structure in the horizontal direction alongside the beams and they realize the optimal control strategies.
Three kinds of dynamic loadings are applied:

(1) Periodic sinusoidal horizontal loading pressure on each joint (Fig. 3 (a)).
(2) Periodic sinusoidal horizontal loading pressure acting on every node on one side of the structure (Fig. 3 (b)).
(3) Periodic impulsive horizontal force acting on each joint on one side of the structure (Fig. 3 (c)).

They correspond to the technical recommendation with \( r_0 = 0.125 \text{N/m}^2; \ v_m = 16.0; \ g = 2.504; \ c_t = 1.0; \ p_w = 0.5r_0v_m^2(1+g)c_ysin(t) \) according to Baniotopoulos and Plalis (2002). The shapes of the loadings are shown in Fig. 3.

The response of the closed-loop system is then compared with the response of the open-loop system with respect to the reduction of the maximum horizontal displacement for the three dynamic loadings. All simulation cases, as it was expected, illustrate asymptotic stability of the three control strategies. Only the data available from the installed actuators are used for the calculation of the LQR control gain as well as for the calculation of the control and filter gains for the \( H_2 \) approach. This leads to heavy penalization of the control in the LQR performance criterion. As expected the maximum displacements for

![Wind Load Diagrams](image_url)

Fig. 3. (a)–(c) Shapes of the considered loadings.
the closed-loop system (controlled structure) are smaller than those of the open-loop system (uncontrolled structure). Fig. 4 displays the horizontal displacement of the eighth floor of the building due to loading (a) employing the three control strategies. The maximum displacement is reduced by 69.5%, 93.1% and 86.2% for the LQR, $H_2$ and $H_\infty$ control laws, respectively. It is observed that the use of the $H_2$ control law leads to more effective vibration response of the structure. Notice that theoretically it is possible to choose a relevant explicitly large value for the weight factor $Q$ and by doing so one can considerably reduce the displacement peak for the LQR control law.

Fig. 5 shows a comparison of the horizontal displacement of the last floor for the uncontrolled structure and the controlled structure. The respective reduction is equal to 74.3%, 93.2% and 95.3%.

Using the three control strategies, the horizontal displacement due to loading (c) is presented in Fig. 6. The maximum magnitude of the horizontal displacement is reduced by 89.4%, 96.6% and 92.3%, respectively. The experiment with loading (c) demonstrates the good transient response of the proposed strategies. In addition, it should be emphasized that $H_2$ and especially $H_\infty$ ensure good robustness of the controlled process. Using $\mu$ analysis for the $H_\infty$ suboptimal controller we obtain that the SSV for the robust stability is equal to 0.802 and satisfies the requirement to be less than one for good robust stability. Also the SSV for robust performance equals 0.867 and it is less than one. Therefore, it guarantees good performance for norm-bounded structured uncertainties $\|\Delta\|_\infty < 1/0.867$.

The good performance for the $H_\infty$ suboptimal controller is corroborated by the fact that the magnitude of the maximal singular value of the closed-loop sensitivity transfer matrix satisfies the inequality (22) and lies under the inverse of the performance weight matrix $W_p$.

5. Conclusions

This paper presents a dynamical model for the active vibration control of a typical multi-storey building structure. The problem of active control is first studied by using the
classical LQR approach. In order to achieve robustness with respect to external disturbances and uncertainties of the system or of the loading, an $H_2$ optimal control problem is investigated. Furthermore, structured uncertainties are considered. The model of the uncertain system is presented in a LFT framework. The $H_{\infty}$ technique is applied in order to avoid the inability of $H_2$ to accommodate directly the considered structured uncertainties. Following current practice in control applications, a suboptimal controller is numerically calculated. The behavior of the closed-loop controlled system for the proposed three controlled strategies is simulated using three kinds of periodic loadings.

Fig. 5. Response of the free (slim solid) and controlled LQR (dot), $H_2$ (bold solid) and $H_{\infty}$ (dash) for the last floor of the structure due to loading (b).

Fig. 6. Response of the free (slim solid) and controlled LQR (dot), $H_2$ (bold solid) and $H_{\infty}$ (dash) for the last floor of the structure due to loading (c).
The comparison between the three proposed control laws shows that all three strategies are effective. The latter two methods are preferred due to the afore-mentioned robustness properties. Further research will concentrate on the correlation of the general LFT technique with concrete, damage-, crack- or fatigue-related uncertainties of the structural system. Moreover, it must be emphasized that the periodic loading which is used for the demonstration of the methodology in this paper can be applied to arbitrary, real or realistic wind records by means of Fourier analysis into sinusoidal components and superposition of the results. A thorough investigation in this direction will demonstrate the effectiveness of the various control schemes on vibration suppression for winds of different spectral content will be reported elsewhere in the future.

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